JET PROPULSION LABORATORY
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INTEROFFICE MEMORANDUM
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To: C. Acton

From: J. G. Williams and W. M. Folkner

Subject: DE418 Moon and Lunar Coordinates

Abstract

This memo discusses the DE418 lunar orbit and orientation angles. The DE418 orbit is compared with DE403 and some qualifications are noted. For coordinate frames rotating with the Moon, two choices are described and the rotation between the two frames is given.

Lunar Ephemeris Considerations

Planetary and lunar ephemeris DE418 (Folkner et al, 2007) was provided to satisfy a request by New Horizons. Consequently, an improved Pluto ephemeris was paramount. Improving Mars and the Moon were secondary goals. A joint lunar and planetary fit to data had not been made for a decade. Because of divergence between the separate lunar and planetary analysis programs in that decade, modifications had to be made to both sets of programs. First attempts at a joint solution uncovered problems which required modifications and changes in procedure. With the deadline approaching, and with the Moon a secondary goal, expedient steps were taken. These included a) not solving for the normal full set of lunar parameters, and b) not iterating the solution/integration. The latter is always advisable when producing a lunar ephemeris. Lack of an iteration may limit the extrapolation of the lunar orbit to future times since the orbital longitude secular tidal acceleration, with a t^2 growth, is sensitive to iteration.

Following the delivery of DE418, fits were made to lunar laser ranges and the prefit residuals are small. Without an iteration, we are hesitant to propose the DE418 lunar ephemeris and Euler angles as a widespread long-term replacement for DE403, but we do expect the DE418 Moon to be an improvement over DE403 for times near the present time. It should be possible to improve on the DE418 Moon and another ephemeris of the Moon and planets is planned for next year.

DE418 vs DE403 Moon

DE403 was generated in 1992 and, compared to DE418, the lunar position is diverging. Figure 1 shows the differences in radius as well as ecliptic longitude and latitude from 1990 to 2020. The longitude difference shows the t^2 behavior from a small change (0.7%) in secular tidal acceleration and the increasing monthly variation in radius difference reflects the same effect on mean anomaly. The half meter bias is due to a difference in the GM of earth+moon. The latitude pattern results from two monthly oscillations beating together while the amplitude of one increases due to the secular acceleration difference. The position difference is typically about 5 m in 2007, increasing to ~8 m in 2011 and ~16 m in 2020. The difference is mainly orthogonal to radius so the corresponding angles are about 3, 4, and 9 mas (milliseconds of arc), respectively.

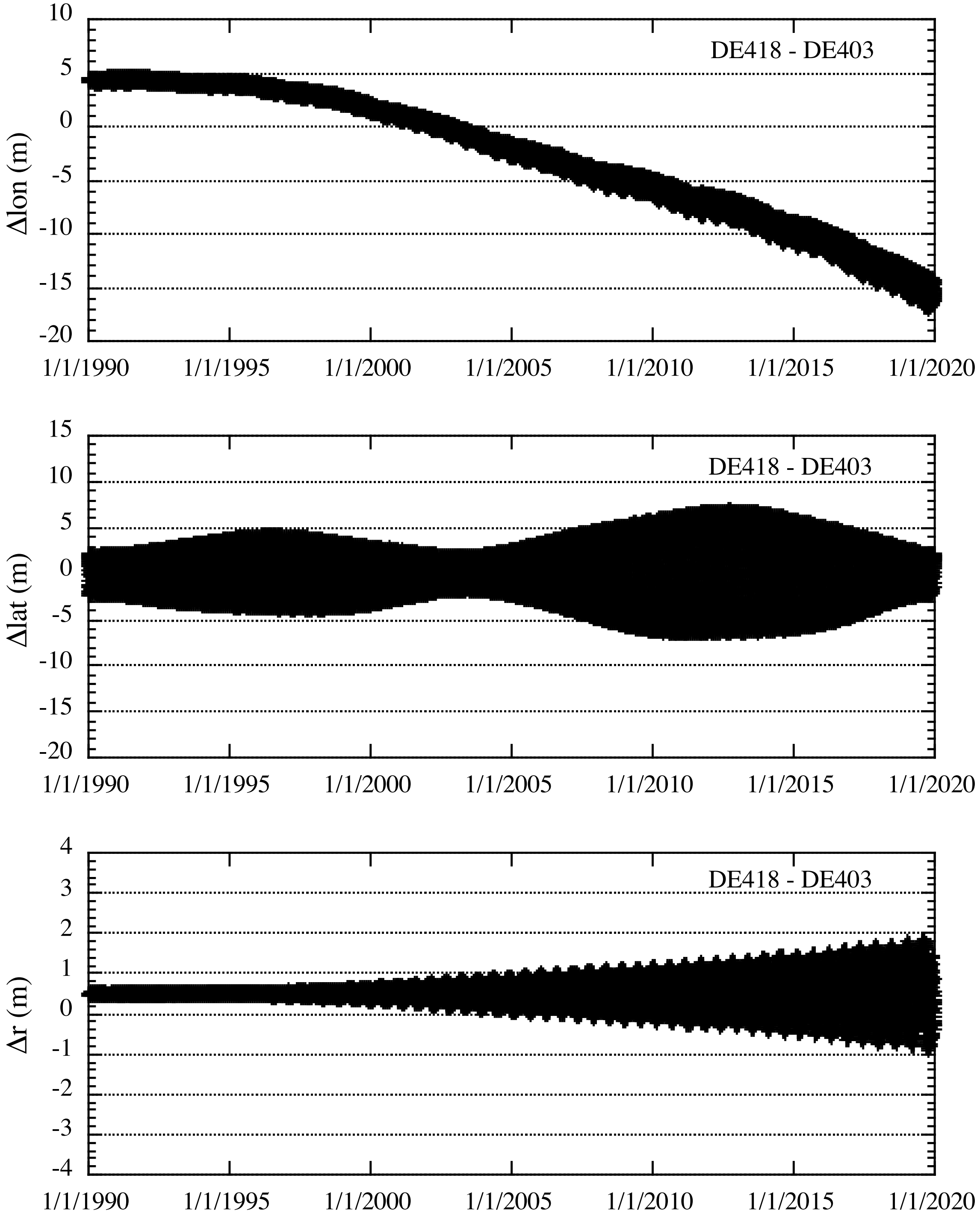

Figure 1. Moon differences for ecliptic longitude and latitude, and radius for DE418 – DE403.

Lunar Surface Coordinate Frames — Principal Axes and Mean Earth/Mean Rotation Axes

The solutions for lunar orientation Euler angles and lunar laser retroreflector array coordinates use the principal axes of the lunar moment of inertia matrix for the X, Y and Z directions. It is natural to write the equations of motion for the Euler angles using the principal axes. The center of mass is used for the origin of Moon-fixed frames.

The other frame of interest uses the mean direction toward the Earth for the X axis and the mean rotation axis for the Z axis. Y completes the right-handed triad. In Lunar Laser Ranging (LLR) papers we have called this the mean earth/mean rotation axis frame or more tersely the mean earth/rotation axis frame. In papers by Mert Davies and the IAU/IAG working group on coordinates and rotations (Seidelmann et al, 2007, and earlier papers in this sequence) this designation has been changed to mean earth/polar axis frame. An ellipsoidal Moon with only a second-degree figure (gravity) would have the mean axis and principal axis frames coincident. It is the third and higher degree representations of the gravity field which cause a constant separation between the two frames (Williams et al, 2006).

A constant three-angle rotation relates the two frames, but our knowledge of the three constant angles depends on the gravity field coefficients and it requires a theory relating them. Since gravity field coefficients change for different ephemerides, the rotation angles must be compatible with the ephemeris. The gravity harmonic coefficients have improved over the years. For DE418, two of the critical third-degree coefficients and all of the fourth-degree coefficients were based on the LP150Q values of Konopliv et al (2001).

The fits to lunar rotation and orbit are done together. The integration for orbit and Euler angle evolution is simultaneous. Thus, the lunar Euler angles (physical librations) and orbit on the DE file are compatible and the Euler angles use the principal axis frame. Note also that the Konopliv et al (2001) LP150Q gravity field uses the principal axis frame, albeit for DE403 with an orientation 4" different. Thus, use of the JPL ephemeris file with the LP150Q gravity field is recommended for high accuracy navigation purposes.

There are no equations of motions for Euler angles referenced to mean earth/mean rotation axes. The Euler angles from space to principal axes that are provided with DE files are the most accurate representation available. For a given gravity field, the constant three-angle rotation from principal axes to mean axes is less accurately known than the integrated Euler angles. The constant rotation is typically computed after the fit and integration. If one wishes the orientation of the lunar mean earth/mean rotation axes, we recommend first extracting the principal axis angles from the file and then rotating by three constant angles.

Note that the semianalytical series expressions for angle W along with pole right ascension and declination, given in the IAU/IAG working group documents (Seidelmann et al, 2007, and earlier), are approximations of much lower accuracy than the integrations. By comparing with integrated values, Konopliv et al (2001) showed that the series expressions for orientation lead to position errors which can exceed 100 m. One of us (JGW) derived the series expressions and that uncertainty is consistent with the level of truncation of the series.

Moon-Centered Coordinates

Rotations between the two lunar coordinate frames was first developed for the Lunar Laser Ranging (LLR) coordinates of retroreflector arrays. The fits to the LLR data use the integrated lunar orbit and physical libration Euler angles, so the derived retroreflector array coordinates are in the principal axis frame. Techniques have been developed for determining the constant rotation between principal axis coordinates and mean earth/mean rotation axis coordinates.

The constant rotation between principal axis coordinates and mean earth/mean rotation axis coordinates depends on the lunar gravity field coefficients and the calculation ultimately rests on a theory. Originally this theory was by Eckhardt (1981) and the relevant terms are the constant parts of p_1, p_2, and tau. The first two of these are Moon-fixed coordinates of the pole normal to the ecliptic plane and the tau refers to the longitude rotation. The constant parts of p_1, p_2, and tau depend on the gravity field; knowledge of the gravity field was poor in earlier times and Eckhardt's constant part of the tau angle is a factor of about three larger than the actual value of the angle! Eckhardt provided partial derivatives with respect to the third-degree gravity coefficients, but since there are nonlinearities in the differential corrections the accuracy is limited at some level. In rotating the Moon-centered coordinates of LLR retroreflector arrays from principal axis to mean earth/mean rotation axis coordinates, LLR first used Eckhardt's constant parts of p_1, p_2, and tau, but later opted for consistency by deriving empirically the rotations between sets of LLR coordinates determined with different integrated ephemerides. Even with the latter procedure, Eckhardt’s theory was used to rotate an earlier set of coordinates which later empirical rotations depended on.

The last LLR array coordinates which used Eckhardt’s theory for the rotation were in Williams, Newhall and Dickey (1987). Subsequent LLR coordinates during the the past two decades were aligned with those mean earth/mean rotation axis coordinates: 1) An internal memo (Williams, Newhall and Standish, 1993) gave a set for DE245 at the time of the Clementine mission. 2) Williams, Newhall and Dickey (1996) published a set compatible with the gravity coefficients in Dickey et al (1994). 3) LLR coordinates using DE403 and the three associated rotations have been distributed by email. These DE403-related coordinates were used and published in a paper by Davies and Colvin (2000) while the associated angles for the rotations are given in Konopliv et al (2001) and Seidelmann et al (2007).

In this memo the procedure is changed again. The LLR principal axis coordinates determined during the solution leading to DE418 are given in Table 1. For the rotation to the mean earth/mean rotation axis frame, the rotation in longitude, which is very sensitive to the gravity field coefficients, was computed from a new theory in order to overcome the limitations of the gravity field that Eckhardt (1981) used. The unpublished new theory balances the C_{22} torque from the displaced X and Y axes against the torques from the third- and fourth-degree coefficients. A small contribution from tidal dissipation (Williams et al, 2001) is added. For the pole direction, a fit of periodic terms to the Euler angle orientations was used (Newhall and Williams, 1997). If M is a vector of Cartesian coordinates in the mean earth/mean rotation axis frame and P is a vector in the principal axis frame, then the derived rotation is

$$M = R_x(-0.27'')\, R_y(-78.62'')\, R_z(-68.00'')\, P \qquad (1)$$

where the angles are in seconds of arc and the rotations are around the body X, Y and Z axes. The inverse rotation is

$$P = R_z(68.00'') \, R_y(78.62'') \, R_x(0.27'') \, M \tag{2}$$

Note that rotation of coordinates of eqs (1) and (2) above and the corresponding rotation of the principal axes have the opposite sense. Rotation of the LLR coordinates of Table 1 to the mean earth/mean rotation axis frame gives the coordinates in Table 2.

Because of the changed procedure for calculating the constant rotations, the mean axis coordinates of Table 2 are rotated when compared with those previously distributed for DE403 and earlier ephemeris Euler angles. That difference is 0.5" in longitude and 0.2" in pole position. At the lunar surface 1" corresponds to 8.4 m, so the displacements are a few meters. The uncertainties in the new rotation angles are estimated to be 0.2" for the Z rotation and 0.1" for the X and Y rotations, about 2 and 1 m, respectively, excluding gravity field uncertainties. Gravity field uncertainties for S_{31} and S_{33} are most important for the longitude rotation. For LP150Q that uncertainty is about 1.0” and future changes in the principal axis direction due to gravity field changes can be of that order. Comparing mean axis coordinates from DE418 and DE403, there is also a shift of 0.7 m along the X axis due partly to a changed orbital semimajor axis and GM(earth+moon) and partly due to different tidal Love numbers.

The LLR range model includes solid-body tides on the Moon. That calculation includes a constant displacement in addition to time variations. The constant tidal displacement, different for each retroreflector array, is not included in Tables 1 and 2. The lunar Love numbers used during the fit were $h_2 = 0.0408$ and $l_2 = 0.0106$ giving few decimeter constant displacements.

Table 1. Lunar Laser retroreflector array coordinates using principal axis frame.

Array	X	Y	Z	R	Longitude	Latitude
	meters	meters	meters	meters	degrees	degrees
Apollo 11	1591967.720	690697.582	21003.638	1735472.709	23.4542890	0.6934417
Apollo 14	1652689.172	-520999.716	-109730.640	1736336.090	-17.4971248	-3.6233164
Apollo 15	1554678.791	98093.593	765005.208	1735477.302	3.6103331	26.1551860
Lunokhod 2	1339364.716	801870.350	756358.738	1734638.994	30.9087270	25.8510029

Table 2. Lunar Laser retroreflector array coordinates using mean earth/mean rotation axis frame.

Array	X	Y	Z	R	Longitude	Latitude
	meters	meters	meters	meters	degrees	degrees
Apollo 11	1591747.819	691222.347	20397.833	1735472.709	23.4730734	0.6734399
Apollo 14	1652818.897	-520454.696	-110361.319	1736336.090	-17.4786479	-3.6441695
Apollo 15	1554937.845	98605.123	764412.712	1735477.302	3.6285068	26.1333957
Lunokhod 2	1339388.485	802310.870	755849.321	1734638.994	30.9221492	25.8323071

Summary

The DE418 lunar position should be an improvement over DE403 at the present time, but there may be limitations when extrapolating years into the future. For high accuracy purposes such as lunar navigation, we recommend use of the JPL ephemeris file with the LP150Q gravity field. Rotation of Moon-centered coordinates between the mean earth/mean rotation axis frame and the principal axis frame can be achieved with eqs (1) and (2).

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