# The Planetary and Lunar Ephemeris DE 421 

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#### Abstract

The planetary and lunar ephemeris DE 421 represents the 'current best estimates' of the orbits of the Moon and planets. The lunar orbit is currently known to sub-meter accuracy though fitting lunar laser ranging data. The orbits of Venus, Earth, and Mars areknown to sub-kilometer accuracy. Because of perturbation of the orbit of Mars by asteroids, frequent updates are needed to maintain the current accuracy into the future decade. Mercury's orbit is determined to an accuracy of several kilometers by radar ranging. The orbits of Jupiter and Saturn are determined to accuracies of tens of kilometers as a result of spacecraft tracking and modern ground-based astrometry. The orbits of Uranus, Neptune, and Pluto are not as well determined. Reprocessing of historical observations is expected to lead to improvements in their orbits in the next several years.


## 1. Introduction

The planetary and lunar ephemeris DE 421 is a significant advance over earlier ephemerides. Compared with DE 418, released in July 2007 (Folkner et al. 2007), the current ephemeris includes additional data, especially range and VLBI measurements of Mars spacecraft; range measurements to the ESA Venus Express spacecraft; and use of current best estimates of planetary masses in the integration process. The lunar orbit is more robust due to an expanded set of lunar geophysical solution parameters, seven additional months of laser ranging data, and complete convergence. DE 421 has been integrated over the time period 1900 to 2050. A longer integration has been deferred until some improvements in the dynamical models have been implemented.

While the lunar orbit in DE 421 is close to that in DE 418, it is a major improvement over the widely distributed DE 405 (Standish 1998). For DE 405 the lunar orbit was not fit in a way consistent with the other planets. Continuing the process used to develop DE $418, \mathrm{DE} 421$ is a combined fit of lunar laser ranging (LLR) and planetary measurements. The DE 421 model is more complete than for DE 418 and has been fully converged, so it is recommended for use by lunar missions.

Also, DE 405 was created in 1995 before the Mars Pathfinder mission in 1997, so the Earth and Mars orbits were largely dependent on range measurements to the Viking landers from 1976 to 1982 augmented by radar range observations with an accuracy of about 1 km . The error in the Earth and Mars orbits in DE 405, is now known to be about 2 km , which was good accuracy in 1997 but much worse than the current sub-kilometer accuracy.

Because of perturbations of the orbit of Mars by asteroids, frequent updates are needed to maintain the current ephemeris accuracy into the future decade. The orbits of Earth and Mars are continually improved through measurements of spacecraft in orbit about Mars. DE 421 incorporates range data through the end of 2007. VLBI observations of Mars spacecraft were resumed in January 2006 to improve the Mars orbit accuracy for the MSL project. VLBI data
through December 2007 have been included in the DE 421 estimate. The Earth and Mars orbit accuracies are expected to be better than 300 m through 2008.

The Venus orbit accuracy has been significantly improved by inclusion of range measurement to the Venus Express spacecraft. Combined with VLBI measurements of Magellan, and one VLBI observation of Venus Express, the Venus orbit accuracy is now about 200 meters.

The orbit of Mercury is currently determined by radar range observations. Since the last radar range point is in 1999, the estimated Mercury orbit has not changed significantly for the past decade. The current orbit accuracy is a few kilometers. Measurements of the Messenger spacecraft are expected to lead to a significant improvement over the next several years.

The orbits of Jupiter and Saturn are determined to accuracies of tens of kilometers using spacecraft tracking and modern ground-based astrometry. The orbit of Saturn is more accurate than that of Jupiter since the Cassini tracking data are more complete and more accurate than previous spacecraft tracking at Jupiter. The orbits of Uranus, Neptune, and Pluto are not as well determined. Reprocessing of historical observations is expected to lead to improvements in their orbits in the next several years.

Below we briefly summarize the dynamical modeling assumptions used in the development of DE 421 and the measurements used in its estimation.

## 2. Planetary Ephemeris Dynamical Modeling

The time coordinate for DE 421 is consistent with the metric used for integration. The coordinate time has been scaled such that at the location of the Earth the coordinate time has no rate relative to atomic time. In a resolution adopted by the International Astronomical Union in 2006 (GA26.3), the time scale TDB (Temps Dynamique Barycentrique, Barycentric Dynamical Time) was defined to be consistent with the JPL ephemeris time. The conversion from atomic time to coordinate time has been done using the formulation of Fairhead and Bretagnon (1990), updated by Fukushima and Irwin (1999) which is consistent, for planetary navigation accuracies, with the simpler approximation given in Moyer (2000).

The axes of the ephemeris are oriented with respect to the International Celestial Reference Frame (ICRF). The Mars spacecraft VLBI measurements serve to tie the ephemeris to the ICRF with accuracy better than 1 milli-ascsecond ( 1 mas $\approx 5$ nanoradian) for the planets with accurate ranges.

For DE 421 the positions of the moon and planets were integrated using a n-body parameterized post-Newtonian metric (Will and Nordtvedt, 1972; Will 1981; Moyer 2000). The PPN parameters $\gamma$ and $\beta$ have been set to 1 , their values in general relativity. Extended body effects for the Earth-Moon system are described in a companion memo (Williams et al. 2008). The oblateness of the Sun has been modeled with $\mathrm{J}_{2}$ set to $2.0 \times 10^{-7}$. Along with the Earth/Moon mass ratio, the mass parameter GM for the Sun, which is by convention a fixed value in units of $\mathrm{AU}^{3} / \mathrm{day}^{2}$, was estimated in units of $\mathrm{km}^{3} / \mathrm{s}^{2}$ by solving for the AU in km in the development of DE 421. The mass parameter of the Earth-Moon system was held fixed to a previous LLR-only estimate. The mass parameters for the other planets (planetary systems for planets with natural satellites) were taken from published values derived from spacecraft tracking data. The mass parameters used for the Sun and planets are given in Table 1.

Table1: Mass parameters of planetary bodies/systems used in DE 421

| Body/System | $\mathrm{GM}\left(\mathrm{km}^{3} / \mathrm{s}^{2}\right)$ | $\mathrm{GM}_{\text {sun }} / \mathrm{GM}_{\text {planet }}$ | Reference |
| :--- | ---: | ---: | :--- |
| Mercury | 22032.090000 | 6023597.400017 | Anderson et al [1987] |
| Venus | 324858.592000 | 408523.718655 | Konopliv et al. [1999] |
| Earth | 398600.436233 | 332946.048166 | See text |
| Mars | 42828.375214 | 3098703.590267 | Konopliv et al. [2006] |
| Jupiter | 126712764.800000 | 1047.348625 | Jacobson [2005] |
| Saturn | 37940585.200000 | 3497.901768 | Jacobson et al. [2006] |
| Uranus | 5794548.600000 | 22902.981613 | Jacobson et al. [1992] |
| Neptune | 6836535.000000 | 19412.237346 | Jacobson et al. [1991] |
| Pluto | 977.000000 | 135836683.767599 | Jacobson et al. [2007] |
| Sun | 4902.800076 | 27068703.185436 | See text |
| Moon | 403503.236310 | 328900.559150 | LLR fit |
| Earth-Moon |  |  | Estimated |

The orbit of the Sun was not integrated in the same way as the other planets. Instead, the position and velocity of the Sun were derived at each integration time step to keep the solar system barycenter (Estabrook 1971) at the center of the coordinate system.

The Newtonian effects of 67 'major' asteroids and 276 'minor' asteroids that introduce the largest perturbations on the orbit of Mars have been included in the integration of the planetary orbits in an iterative manner. The orbits of Ceres, Pallas, and Vesta were integrated simultaneously, including mutual interactions, holding the orbits of the Sun and planets to those in DE 405. The orbits for the other asteroids were integrated individually under the gravitational forces from the Sun, planets, and Ceres, Pallas and Vesta, whose orbits were held fixed. The mass parameters of Ceres, Pallas, and Vesta, and eight other asteroids were then estimated in fitting the DE 421 data. The mass parameters of the remaining 56 'major' asteroids were held at assumed nominal values. The mass parameters of the major asteroids are given in Appendix A. The minor asteroids were divided into three taxonomic types (classes). The volume of each minor asteroid was based on a nominal radius and the density of each of the three types of asteroids was estimated. The estimated densities and the radii assumed for the minor asteroids are given in Appendix A.

The selection of which asteroid mass parameters to estimate was based on an empirical process to see which set produced a reasonably accurate prediction of the Earth-Mars range over one year. For example, Figure 1 shows Mars Odyssey range residuals relative to DE 418, which was fit to range data through end of 2006. DE 418 is seen to predict range to Mars one year into the future with an accuracy of about 15 m . Similarly DE 421 is expected to predict the EarthMars range to about 15 m through end of 2008. (The error in the plane-of-sky position of Mars relative to Earth through end of 2008 is about 300 m .) This is relevant for navigation of the Phoenix spacecraft with arrival at Mars in May 2008. The estimated mass parameters of the selected asteroids and estimated asteroid class densities are not necessarily the best possible values for other purposes.


Figure 1: Mars Odyssey spacecraft range measurement residuals to planetary ephemeris DE 418. DE 418 was fit to range measurements through end of 2006. The range residuals for data in 2007 (in shaded area) are less than 15 m and indicate the ephemeris prediction accuracy.

## 3. Measurement Set

Rather than try to fit all available planetary observations, the data used for DE 421 were preferentially selected for the best accuracy and (for angular data) accuracy of ties to the ICRF. The measurements are summarized in Table 2 and Table 3. Plots of the residuals for all data are included in Appendix B. The data for each planet contain primary data that have the most strength for determining the orbit, and, for some planets, secondary data that are included in the fit at their nominal weight but do not affect the orbit significantly.

Lunar laser ranging, spacecraft ranging, and radar ranging are all very accurate and independent of reference frame. VLBI observations of spacecraft in orbit about Venus, Mars, Jupiter, and Saturn relative to extragalactic radio sources defining the ICRF tie the planetary ephemeris to the ICRF.

Analysis of spacecraft range and Doppler observations taken as spacecraft fly by planets can give right ascension and declination with accuracy somewhat less than the VLBI observations. These right ascension and declination determinations are important in refining the orbits of Jupiter and Saturn. The accuracy of spacecraft plane-of-sky determinations is very much a function of time. The earliest planetary encounters relied on S -band ( 2 GHz ) radio systems with range and Doppler measurement accuracy very sensitive to electrons in the solar plasma. Later spacecraft observations (e.g. after 1990) used X-band ( 8 GHz ) radio systems that were much less affected by solar plasma. Early spacecraft encounter data were processed with reference frame models not well linked to the current ICRF, and often saw discrepancies between range and Doppler data. Data from most encounters have since been re-processed with modern reference frame models so the determined plane-of-sky positions are consistent with the ICRF. For each encounter a single vector for range, right ascension, and declination was generated. For Cassini, a vector was generated for each orbit about Saturn.

Astrometric observations of the planets in the past have suffered from difficulty in establishing an accurate celestial reference frame. Since the release of the Hipparcos star catalog, and the development of techniques for using CCD instruments, astrometric accuracies are approaching spacecraft VLBI accuracies. However these observations only cover a fraction of the orbital periods of the outer planets. Since the orbits of Jupiter and Saturn are well determined from spacecraft data, the limited time span of modern data mainly affects the orbital uncertainties of Uranus, Neptune, and Pluto. The Pluto data set was discussed in detail in relation to the ephemeris DE 418 (Folkner et al. 2007). For the orbit of Pluto in DE421, we followed the
same approach used for DE 418, with two more months of observations. For Uranus and Neptune the assessment of older data sets is not as complete as for Pluto so relatively few data have been included. These orbits are reasonably accurate for the current times due to modern astrometry and knowledge from the Voyager encounters. The Uranus and Neptune data sets will be expanded in a future ephemeris.

Most of the data used are not published but communicated to the authors electronically. Most data are available at the web site http://iau-comm4.jpl.nasa.gov/plan-eph-data/ or by request from the authors. Lunar laser ranging data are posted by the International Laser Ranging Service (Pearlman et al 2002; http://ilrs.gsfc.nasa.gov/). A Mariner 10 range to Mercury was reported by Anderson et al. (1987). A Goldstone radar range to Mercury is from Jurgens et al. (1998). Radar ranges to Mercury and Venus from Eupatoria are from Kotelnikov et al (1983; http://www.ipa.nw.ru/PAGE/DEPFUND/LEA/ENG/rrr.html). Astrometry data from the US Naval Observatory are from Stone et al. (2003; http://www.nofs.navy.mil/data/plansat.html). Older observations from Pluto are taken from the literature (see references: Barbieri, Cohen, Jenson, Gemmo, Klemola, Rapaport, Rylkov, Sharaf, Zappala). Other data were via private communications.

## 4. Availability

The DE421 ephemeris may be downloaded in an ascii version from
ftp://ssd.jpl.nasa.gov/pub/eph/planets/ascii/de421 .
The complete set of input parameters for the solar system integration is part of the file. The SPICE kernal version of DE421 is available at
ftp://ssd.jpl.nasa.gov/pub/eph/planets/bsp

Table 2: Summary of data used to estimate orbits of the Moon, inner planets and Jupiter. Data with relatively little contribution to the estimates orbits are indicated in italics.

| Planet | Meas | Type | Obs | Span | \#meas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Moon | LLR | range |  |  |  |
|  |  |  | McDonald 2.7m | 1970-1985 | 3451 |
|  |  |  | MLRS/saddle | 1984-1988 | 275 |
|  |  |  | MLRS/MtFowlkes | 1988-2007 | 2746 |
|  |  |  | Haleakala | 1984-1990 | 694 |
|  |  |  | CERGA | 1984-2005 | 9177 |
|  |  |  | Matera | 2004 | 11 |
|  |  |  | Apache Pt | 2006-2007 | 247 |
| Mercury | Radar | range | Arecibo | 1967-1982 | 242 |
|  |  |  | Goldstone | 1972-1997 | 283 |
|  |  |  | Haystack | 1966-1971 | 217 |
|  |  |  | Eupatoria | 1980-1995 | 75 |
|  | Radar | closure | Goldstone | 1989-1997 | 40 |
|  | Spacecraft | range | Mariner 10 | 1974-1975 | 2 |
| Venus | Spacecraft | range | VEX | 2006-2007 | 14304 |
|  | Spacecraft | VLBI | VEX | 2007 | 1 |
|  | Spacecraft | VLBI | MGN | 1990-1994 | 18 |
|  | Spacecraft | 3-D | Cassini | 1998-1999 | 2 |
|  | Radar | range | Arecibo | 1967-1970 | 227 |
|  |  |  | Goldstone | 1970-1990 | 512 |
|  |  |  | Haystack | 1966-1971 | 229 |
|  |  |  | Millstone | 1964-1967 | 101 |
|  |  |  | Eupatoria | 1962-1995 | 1134 |
| Mars | Spacecraft | range | Viking L1 | 1976-1982 | 1178 |
|  |  |  | Viking L2 | 1976-1977 | 80 |
|  |  |  | Pathfinder | 1997 | 90 |
|  |  |  | MGS | 1999-2006 | 164781 |
|  |  |  | Odyssey | 2002-2007 | 251999 |
|  |  |  | MEX | 2005-2007 | 63133 |
|  |  |  | MRO | 2006-2007 | 7972 |
|  | Spacecraft | VLBI | MGS | 2001-2003 | 14 |
|  |  |  | ODY | 2002-2007 | 66 |
|  |  |  | MRO | 2006-2007 | 14 |
| Jupiter | Spacecraft | $3-\mathrm{D}$ | Pioneer 10 | 1973 | 1 |
|  |  |  | Pioneer 11 | 1974 | 1 |
|  |  |  | Voyager 1 | 1979 | 1 |
|  |  |  | Voyager 2 | 1979 | 1 |
|  |  |  | Ulysses | 1992 | 1 |
|  |  |  | Cassini | 2000 | 1 |
|  | CCD | ra/dec | USNOFS | 1998-2007 | 2533 |
|  | Spacecraft | VLBI | Galileo | 1996-1997 | 24 |
|  | Transit | ra/dec | Washington | 1914-1994 | 2053 |
|  |  |  | Herstmonceux | 1958-1982 | 468 |
|  |  |  | La Palma | 1992-1997 | 658 |
|  |  |  | Tokyo | 1986-1988 | 98 |
|  |  |  | El Leoncito | 1998 | 11 |

Table 3: Summary of data used to estimate orbits of Saturn, Uranus, Neptune, and Pluto. Data with relatively little contribution to the estimates orbits are indicated in italics.

| Planet | Meas | Type | Obs | Span | \#meas |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Saturn | Spacecraft | 3-D | Pioneer 11 | 1979 | 1 |
|  |  |  | Voyager 1 | 1980 | 1 |
|  |  |  | Voyager 2 | 1981 | 1 |
|  |  |  | Cassini | 2004-2006 | 31 |
|  | CCD | ra/dec | USNOFS | 1998-2007 | 3153 |
|  |  |  | TMO | 2002-2005 | 778 |
|  | Transit | ra/dec | Bordeaux | 1987-1993 | 119 |
|  |  |  | Washington | 1913-1982 | 1422 |
|  |  |  | Herstmonceux | 1958-1982 | 405 |
|  |  |  | La Palma | 1992-1997 | 730 |
|  |  |  | Tokyo | 1986-1988 | 62 |
|  |  |  | El Leoncito | 1998 | 18 |
| Uranus | Spacecraft | 3-D | Voyager 2 | 1986 | 1 |
|  | CCD | ra/dec | USNOFS | 1998-2007 | 1612 |
|  |  |  | TMO | 1998-2007 | 347 |
|  | Transit | ra/dec | Bordeaux | 1985-1993 | 165 |
|  |  |  | Washington | 1914-1993 | 2043 |
|  |  |  | Herstmonceux | 1957-1981 | 353 |
|  |  |  | La Palma | 1984-1997 | 1030 |
|  |  |  | Tokyo | 1986-1988 | 44 |
|  |  |  | El Leoncito | 1997-1998 | 8 |
| Neptune | Spacecraft | 3-D | Voyager 2 | 1989 | 1 |
|  | CCD | ra/dec | USNOFS | 1998-2007 | 1588 |
|  |  |  | TMO | 2001-2007 | 267 |
|  | Transit | ra/dec | Bordeaux | 1985-1993 | 348 |
|  |  |  | Washington | 1913-1993 | 1838 |
|  |  |  | Herstmonceux | 1958-1981 | 316 |
|  |  |  | La Palma | 1984-1998 | 1106 |
|  |  |  | Tokyo | 1986-1988 | 59 |
|  |  |  | El Leoncito | 1998-1999 | 11 |
| Pluto | CCD | ra/dec | USNOFS | 1998-2007 | 852 |
|  |  |  | TMO | 2001-2007 | 118 |
|  | Photo | ra/dec | misc | 1914-1958 | 42 |
|  |  |  | Palomar | 1963-1965 | 8 |
|  |  |  | Pulkovo | 1930-1992 | 53 |
|  |  |  | Bord/Valin | 1995-2001 | 97 |
|  |  |  | Asiago | 1969-1989 | 193 |
|  |  |  | Copenhagen | 1975-1978 | 15 |
|  |  |  | Lick | 1980-1985 | 11 |
|  |  |  | Torino | 1973-1982 | 37 |
|  | Transit | ra/dec | La Palma | 1989-1998 | 380 |
|  |  |  | El Leoncito | 1999 | 33 |

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## Appendix A: Asteroid Parameters

Table A1: Parameters of 'major' asteroids: type; radius r in km ; mass parameter GM in $\mathrm{km}^{3} / \mathrm{s}^{2}$; density $\rho$ in $\mathrm{gm} / \mathrm{cm}^{3}$.

| ID | Name | r | Type |  |
| :--- | ---: | ---: | ---: | ---: |
| GM | $\rho$ |  |  |  |
| 1 Ceres | 474.0 | G | 62.178 | 2.1 |
| 2 Pallas | 266.0 | B | 13.402 | 2.5 |
| 3 Juno | 117.0 | Sk | 1.536 | 3.4 |
| 4 Vesta | 265.0 | V | 17.630 | 3.4 |
| 5 Astraea | 59.5 | S | 0.159 | 2.7 |
| 6 Hebe | 92.6 | S | 0.605 | 2.7 |
| 7 Iris | 99.9 | S | 0.796 | 2.9 |
| 8 Flora | 67.9 | S | 0.236 | 2.7 |
| 9 Metis | 95.0 | S | 0.567 | 2.4 |
| 10 Hygiea | 203.6 | C | 5.364 | 2.3 |
| 11 Parthenope | 77.7 | S | 0.356 | 2.7 |
| 13 Egeria | 103.8 | C | 0.412 | 1.3 |
| 14 Irene | 76.0 | S | 0.348 | 2.8 |
| 15 Eunomia | 127.7 | S | 1.638 | 2.8 |
| 16 Psyche | 126.6 | M | 2.233 | 3.9 |
| 18 Melpomene | 70.3 | S | 0.267 | 2.7 |
| 19 Fortuna | 100.0 | Ch | 0.463 | 1.7 |
| 20 Massalia | 72.8 | S | 0.291 | 2.7 |
| 21 Lutetia | 47.9 | M | 0.139 | 4.5 |
| 22 Kalliope | 90.5 | M | 0.491 | 2.4 |
| 23 Thalia | 53.8 | S | 0.129 | 3.0 |
| 24 Themis | 99.0 | C | 0.403 | 1.5 |
| 25 Phocaea | 37.6 | S | 0.040 | 2.7 |
| 27 Euterpe | 48.0 | S | 0.084 | 2.7 |
| 28 Bellona | 60.5 | S | 0.165 | 2.7 |
| 29 Amphitrite | 106.1 | S | 0.906 | 2.7 |
| 30 Urania | 49.8 | S | 0.095 | 2.7 |
| 31 Euphrosyne | 128.0 | C | 1.139 | 1.9 |
| 41 Daphne | 87.0 | Ch | 0.527 | 2.9 |
| 42 Isis | 50.1 | S | 0.092 | 2.6 |
| 45 Eugenia | 107.3 | C | 0.397 | 1.2 |
| 51 Nemausa | 73.9 | C | 0.144 | 1.3 |
| 52 Europa | 151.3 | C | 1.354 | 1.4 |
| 60 Echo | 30.1 | S | 0.021 | 2.7 |


| ID Name | r | Type | GM | $\rho$ |
| :---: | :---: | :---: | :---: | :---: |
| 63 Ausonia | 51.6 | S | 0.102 | 2.7 |
| 65 Cybele | 118.6 | C | 0.694 | 1.5 |
| 69 Hesperia | 69.1 | M | 0.414 | 4.5 |
| 78 Diana | 60.3 | C | 0.085 | 1.4 |
| 94 Aurora | 102.4 | C | 0.414 | 1.4 |
| 97 Klotho | 41.4 | M | 0.089 | 4.5 |
| 98 Ianthe | 52.2 | C | 0.055 | 1.4 |
| 105 Artemis | 59.5 | C | 0.088 | 1.5 |
| 111 Ate | 67.3 | C | 0.116 | 1.4 |
| 135 Hertha | 39.6 | M | 0.078 | 4.5 |
| 139 Juewa | 78.3 | C | 0.188 | 1.4 |
| 145 Adeona | 75.6 | C | 0.151 | 1.3 |
| 187 Lamberta | 65.6 | C | 0.105 | 1.3 |
| 192 Nausikaa | 51.6 | S | 0.107 | 2.8 |
| 194 Prokne | 84.2 | C | 0.182 | 1.1 |
| 216 Kleopatra | 62.0 | M | 0.299 | 4.5 |
| 230 Athamantis | 54.5 | S | 0.126 | 2.8 |
| 324 Bamberga | 114.5 | CP | 0.661 | 1.6 |
| 337 Devosa | 29.6 | M | 0.033 | 4.5 |
| 344 Desiderata | 66.1 | C | 0.114 | 1.4 |
| 354 Eleonora | 77.6 | Sl | 0.327 | 2.5 |
| 372 Palma | 94.3 | C | 0.355 | 1.5 |
| 405 Thia | 62.5 | C | 0.092 | 1.4 |
| 409 Aspasia | 80.8 | C | 0.216 | 1.5 |
| 419 Aurelia | 64.5 | C | 0.102 | 1.4 |
| 451 Patientia | 112.5 | C | 0.610 | 1.5 |
| 488 Kreusa | 75.1 | C | 0.164 | 1.4 |
| 511 Davida | 163.0 | C | 1.638 | 1.4 |
| 532 Herculina | 111.1 | S | 0.886 | 2.3 |
| 554 Peraga | 47.9 | C | 0.044 | 1.4 |
| 654 Zelinda | 63.7 | Ch | 0.090 | 1.2 |
| 704 Interamnia | 158.3 | C | 2.464 | 2.2 |
| 747 Winchester | 85.9 | C | 0.196 | 1.1 |

Table A2: Estimated densities $\rho$ in $\mathrm{gm} / \mathrm{cm}^{3}$ of 'minor' asteroids

| Type | $\rho$ |
| :---: | :---: |
| C | 1.093 |
| S | 3.452 |
| M | 4.221 |

Table A3: Parameters of 'minor' asteroids: type and radius r in km .
(Continued on next page.)

| ID Name | Type | r |
| :---: | :---: | :---: |
| 326 Tamara | C | 46.5 |
| 328 Gudrun | S | 61.5 |
| 329 Svea | C | 38.9 |
| 334 Chicago | C | 77.9 |
| 335 Roberta | c | 44.5 |
| 336 Lacadiera | C | 34.6 |
| 338 Budrosa | M | 31.6 |
| 345 Tercidina | C | 47.1 |
| 346 Hermentari, | S | 53.3 |
| 347 Pariana | M | 25.6 |
| 349 Dembowska | S | 69.9 |
| 350 Ornamenta | C | 59.2 |
| 356 Liguria | C | 65.7 |
| 357 Ninina | C | 53.0 |
| 358 Apollonia | C | 44.7 |
| 360 Carlova | C | 57.9 |
| 362 Havnia | C | 49.0 |
| 363 Padua | C | 48.5 |
| 365 Corduba | C | 53.0 |
| 366 Vincentina | C | 46.9 |
| 369 Aeria | M | 30.0 |
| 373 Melusina | C | 47.9 |
| 375 Ursula | c | 108.0 |
| 377 Campania | C | 45.5 |
| 381 Myrrha | C | 60.3 |
| 385 Ilmatar | S | 45.8 |
| 386 Siegena | C | 82.5 |
| 387 Aquitania | S | 50.3 |
| 388 Charybdis | C | 57.1 |
| 389 Industria | S | 39.5 |
| 393 Lampetia | C | 48.4 |
| 404 Arsinoe | C | 48.8 |
| 407 Arachne | C | 47.5 |
| 410 Chloris | C | 61.8 |
| 412 Elisabetha | C | 45.5 |
| 415 Palatia | C | 38.2 |
| 416 Vaticana | S | 42.7 |
| 420 Bertholda | C | 70.6 |
| 423 Diotima | C | 104.4 |
| 424 Gratia | C | 43.6 |
| 426 Hippo | C | 63.5 |
| 431 Nephele | C | 47.5 |
| 432 Pythia | S | 23.4 |
| 433 Eros | S | 9.7 |
| 442 Eichsfeldi، | C | 32.9 |
| 444 Gyptis | C | 79.8 |


| ID Name | Type | r |
| :---: | :---: | :---: |
| 445 Edna | C | 43.6 |
| 449 Hamburga | C | 42.8 |
| 454 Mathesis | C | 40.8 |
| 455 Bruchsalia | C | 42.2 |
| 464 Megaira | C | 37.0 |
| 465 Alekto | C | 36.7 |
| 466 Tisiphone | c | 57.8 |
| 469 Argentina | C | 62.8 |
| 471 Papagena | S | 67.1 |
| 476 Hedwig | C | 58.4 |
| 481 Emita | C | 116.0 |
| 485 Genua | S | 31.9 |
| 489 Comacina | C | 69.7 |
| 490 Veritas | C | 57.8 |
| 491 Carina | C | 48.7 |
| 498 Tokio | C | 41.4 |
| 503 Evelyn | C | 40.8 |
| 505 Cava | C | 57.5 |
| 506 Marion | C | 53.0 |
| 508 Princetonia | C | 71.2 |
| 514 Armida | C | 53.1 |
| 516 Amherstia | M | 36.5 |
| 517 Edith | M | 45.6 |
| 521 Brixia | C | 57.8 |
| 535 Montague | C | 37.2 |
| 536 Merapi | C | 75.7 |
| 545 Messalina | C | 55.6 |
| 547 Praxedis | M | 34.8 |
| 566 Stereoskopi. | C | 84.1 |
| 568 Cheruskia | C | 43.5 |
| 569 Misa | C | 36.5 |
| 584 Semiramis | S | 27.0 |
| 585 Bilkis | C | 29.1 |
| 591 Irmgard | M | 25.9 |
| 593 Titania | C | 37.7 |
| 595 Polyxena | C | 54.5 |
| 596 Scheila | C | 56.7 |
| 598 Octavia | C | 36.2 |
| 599 Luisa | S | 32.4 |
| 602 Marianna | C | 62.4 |
| 604 Tekmessa | M | 32.6 |
| 618 Elfriede | C | 60.1 |
| 623 Chimaera | C | 22.1 |
| 626 Notburga | C | 50.4 |
| 635 Vundtia | C | 49.1 |
| 663 Gerlinde | C | 50.4 |


| ID Name | Type | r |
| :---: | :---: | :---: |
| 667 Denise | C | 40.6 |
| 674 Rachele | S | 48.7 |
| 675 Ludmilla | S | 38.0 |
| 680 Genoveva | C | 42.0 |
| 683 Lanzia | C | 41.0 |
| 690 Wratislavi, | C | 67.5 |
| 691 Lehigh | C | 43.8 |
| 694 Ekard | C | 45.4 |
| 696 Leonora | C | 37.9 |
| 702 Alauda | c | 97.4 |
| 705 Erminia | C | 67.1 |
| 709 Fringilla | C | 48.3 |
| 712 Boliviana | C | 63.8 |
| 713 Luscinia | c | 52.8 |
| 735 Marghanna | C | 37.2 |
| 739 Mandeville | C | 53.7 |
| 740 Cantabia | C | 45.4 |
| 751 Faina | C | 55.3 |
| 752 Sulamitis | M | 31.4 |
| 760 Massinga | S | 35.6 |
| 762 Pulcova | c | 68.5 |
| 769 Tatjana | c | 53.2 |
| 772 Tanete | C | 58.8 |
| 773 Irmintraud | C | 47.9 |
| 776 Berbericia | C | 75.6 |
| 778 Theobalda | C | 32.0 |
| 780 Armenia | C | 47.2 |
| 784 Pickeringi, | C | 44.7 |
| 786 Bredichina | C | 45.8 |
| 788 Hohenstein. | C | 51.8 |
| 790 Pretoria | C | 85.2 |
| 791 Ani | C | 51.8 |
| 804 Hispania | C | 78.6 |
| 814 Tauris | C | 54.8 |
| 849 Ara | M | 30.9 |
| 895 Helio | C | 71.0 |
| 909 Ulla | C | 58.2 |
| 914 Palisana | C | 38.3 |
| 980 Anacostia | S | 43.1 |
| 1015 Christa | c | 48.5 |
| 1021 Flammario | C | 49.7 |
| 1036 Ganymed | S | 15.8 |
| 1093 Freda | C | 58.4 |
| 1107 Lictoria | M | 39.6 |
| 1171 Rusthaweli, | c | 35.1 |
| 1467 Mashona | C | 112.0 |

Table A3 (continued): Parameters of 'minor' asteroids: type and radius r in km .

| ID Name | Type | r |
| :---: | :---: | :---: |
| 326 Tamara | C | 46.5 |
| 328 Gudrun | S | 61.5 |
| 329 Svea | c | 38.9 |
| 334 Chicago | C | 77.9 |
| 335 Roberta | c | 44.5 |
| 336 Lacadiera | C | 34.6 |
| 338 Budrosa | M | 31.6 |
| 345 Tercidina | C | 47.1 |
| 346 Hermentari, | S | 53.3 |
| 347 Pariana | M | 25.6 |
| 349 Dembowska | S | 69.9 |
| 350 Ornamenta | C | 59.2 |
| 356 Liguria | c | 65.7 |
| 357 Ninina | C | 53.0 |
| 358 Apollonia | C | 44.7 |
| 360 Carlova | C | 57.9 |
| 362 Havnia | C | 49.0 |
| 363 Padua | C | 48.5 |
| 365 Corduba | C | 53.0 |
| 366 Vincentina | C | 46.9 |
| 369 Aeria | M | 30.0 |
| 373 Melusina | C | 47.9 |
| 375 Ursula | C | 108.0 |
| 377 Campania | C | 45.5 |
| 381 Myrrha | C | 60.3 |
| 385 Ilmatar | S | 45.8 |
| 386 Siegena | C | 82.5 |
| 387 Aquitania | S | 50.3 |
| 388 Charybdis | C | 57.1 |
| 389 Industria | S | 39.5 |
| 393 Lampetia | C | 48.4 |
| 404 Arsinoe | C | 48.8 |
| 407 Arachne | C | 47.5 |
| 410 Chloris | C | 61.8 |
| 412 Elisabetha | C | 45.5 |
| 415 Palatia | C | 38.2 |
| 416 Vaticana | S | 42.7 |
| 420 Bertholda | c | 70.6 |
| 423 Diotima | C | 104.4 |
| 424 Gratia | c | 43.6 |
| 426 Hippo | C | 63.5 |
| 431 Nephele | C | 47.5 |
| 432 Pythia | S | 23.4 |
| 433 Eros | S | 9.7 |
| 442 Eichsfeldi. | C | 32.9 |
| 444 Gyptis | C | 79.8 |


| ID Name | Type | r |
| :---: | :---: | :---: |
| 445 Edna | C | 43.6 |
| 449 Hamburga | C | 42.8 |
| 454 Mathesis | C | 40.8 |
| 455 Bruchsalia | C | 42.2 |
| 464 Megaira | C | 37.0 |
| 465 Alekto | C | 36.7 |
| 466 Tisiphone | C | 57.8 |
| 469 Argentina | C | 62.8 |
| 471 Papagena | S | 67.1 |
| 476 Hedwig | c | 58.4 |
| 481 Emita | C | 116.0 |
| 485 Genua | S | 31.9 |
| 489 Comacina | C | 69.7 |
| 490 Veritas | C | 57.8 |
| 491 Carina | C | 48.7 |
| 498 Tokio | C | 41.4 |
| 503 Evelyn | C | 40.8 |
| 505 Cava | C | 57.5 |
| 506 Marion | C | 53.0 |
| 508 Princetonia | C | 71.2 |
| 514 Armida | C | 53.1 |
| 516 Amherstia | M | 36.5 |
| 517 Edith | M | 45.6 |
| 521 Brixia | C | 57.8 |
| 535 Montague | c | 37.2 |
| 536 Merapi | C | 75.7 |
| 545 Messalina | C | 55.6 |
| 547 Praxedis | M | 34.8 |
| 566 Stereoskopia | C | 84.1 |
| 568 Cheruskia | C | 43.5 |
| 569 Misa | C | 36.5 |
| 584 Semiramis | S | 27.0 |
| 585 Bilkis | C | 29.1 |
| 591 Irmgard | M | 25.9 |
| 593 Titania | C | 37.7 |
| 595 Polyxena | C | 54.5 |
| 596 Scheila | C | 56.7 |
| 598 Octavia | C | 36.2 |
| 599 Luisa | S | 32.4 |
| 602 Marianna | C | 62.4 |
| 604 Tekmessa | M | 32.6 |
| 618 Elfriede | c | 60.1 |
| 623 Chimaera | C | 22.1 |
| 626 Notburga | C | 50.4 |
| 635 Vundtia | C | 49.1 |
| 663 Gerlinde | C | 50.4 |


| ID Name | Type | $r$ |
| :---: | :---: | :---: |
| 667 Denise | C | 40.6 |
| 674 Rachele | S | 48.7 |
| 675 Ludmilla | S | 38.0 |
| 680 Genoveva | c | 42.0 |
| 683 Lanzia | C | 41.0 |
| 690 Wratislavi, | C | 67.5 |
| 691 Lehigh | C | 43.8 |
| 694 Ekard | C | 45.4 |
| 696 Leonora | C | 37.9 |
| 702 Alauda | C | 97.4 |
| 705 Erminia | C | 67.1 |
| 709 Fringilla | C | 48.3 |
| 712 Boliviana | C | 63.8 |
| 713 Luscinia | C | 52.8 |
| 735 Marghanna | C | 37.2 |
| 739 Mandeville | C | 53.7 |
| 740 Cantabia | C | 45.4 |
| 751 Faina | C | 55.3 |
| 752 Sulamitis | M | 31.4 |
| 760 Massinga | S | 35.6 |
| 762 Pulcova | C | 68.5 |
| 769 Tatjana | C | 53.2 |
| 772 Tanete | C | 58.8 |
| 773 Irmintraud | C | 47.9 |
| 776 Berbericia | C | 75.6 |
| 778 Theobalda | C | 32.0 |
| 780 Armenia | C | 47.2 |
| 784 Pickeringi, | C | 44.7 |
| 786 Bredichina | C | 45.8 |
| 788 Hohenstein | C | 51.8 |
| 790 Pretoria | C | 85.2 |
| 791 Ani | C | 51.8 |
| 804 Hispania | C | 78.6 |
| 814 Tauris | C | 54.8 |
| 849 Ara | M | 30.9 |
| 895 Helio | C | 71.0 |
| 909 Ulla | C | 58.2 |
| 914 Palisana | C | 38.3 |
| 980 Anacostia | S | 43.1 |
| 1015 Christa | C | 48.5 |
| 1021 Flammario | C | 49.7 |
| 1036 Ganymed | S | 15.8 |
| 1093 Freda | C | 58.4 |
| 1107 Lictoria | M | 39.6 |
| 1171 Rusthaweli, | C | 35.1 |
| 1467 Mashona | C | 112.0 |

## Appendix B. Measurement Residual Plots



Figure B-1: Lunar laser ranging residuals.


Figure B-2: Mercury radar range residuals.


Figure B-3: a) Mercury radar closure residuals. b) Mariner 10 range residuals at Mercury


Figure B-4: Venus Express range residuals.


Figure B-5: Venus spacecraft VLBI residuals: a) Magellan from Goldstone-Canberra baseline; b) Magellan from Goldstone-Madrid baseline; c) Venus Express from Goldstone-Madrid baseline.


Figure B-6: Venus radar range residuals.


Figure B-7: Residuals for Cassini encounters at Venus.


Figure B-8: Residuals for Mars spacecraft VLBI on Goldstone-Canberra baseline.


Figure B-9: Residuals for Mars spacecraft VLBI on Goldstone-Madrid baseline.



Figure B-11: Post-Viking Mars spacecraft range residuals.


Figure B-12: Jupiter right ascension from spacecraft encounters.


Figure B-13: Jupiter declination from spacecraft encounters.


Figure B-14: Earth-Jupiter range from spacecraft encounters.


Figure B-15: VLBI observations of Galileo at Jupiter on a) Goldstone-Canberra baseline and b) Goldstone-Madrid baseline.


Figure B-16: Observations of Galilean satellites from US Naval Observatory, Flagstaff.


Figure B-17: Transit observations of Jupiter


Figure B-18: Saturn right ascension from Voyage 1\&2 and Cassini tracking analysis.


Figure B-19: Saturn declination from spacecraft encounters.


Figure B-20: Saturn-Earth range from spacecraft encounters.


Figure B-21: Saturn satellite (3-6) observations from US Naval Observatory, Flagstaff.


Figure B-22: Saturn satellite (7-9) observations from US Naval Observatory, Flagstaff.









Figure B-23: Saturn satellite (1-4) observations from Table Mountain Observatory.


Figure B-24: Saturn satellite (5-8) observations from Table Mountain Observatory.


Figure B-25: Saturn satellite (9) observations from Table Mountain Observatory.



Figure B-26: Transit observations of Saturn


Figure B-27: Uranus right ascension, declination, and range from Voyager 2 encounter.


Figure B-28: Uranus observations from US Naval Observatory, Flagstaff.






Figure B-29: Uranus observations from Table Mountain Observatory.


Figure B-30: Transit observations of Uranus


Figure B-31: Neptune right ascension, declination, and range from Voyager 2 encounter.


Figure B-32: Neptune observations from US Naval Observatory, Flagstaff.


Figure B-33: Transit observations of Neptune


Figure B-34: Neptune observations from Table Mountain Observatory.



Figure B-35: Residuals of modern Pluto observations.


Figure B-36: Residuals of Pluto observations made 1968-1990.


Figure B-37: Residuals of Pluto normalized points from Lowell, Yerkes, McDonald, etc.



Figure B-38: Residuals of Pluto observations from Pulkovo astrograph.


Figure B-39: Residuals of Pluto pre-discovery observations.


Figure B-40: Transit observations of Pluto

